Aligned neutron-proton pairs in N=Z nuclei

P. Van Isacker, GANIL, France

Motivation

Shell-model analysis

A model with high-spin bosons

Experimental tests

Brix Day, ULB, Brussels, 2 December 2013

Spin-aligned T=0 np pairs

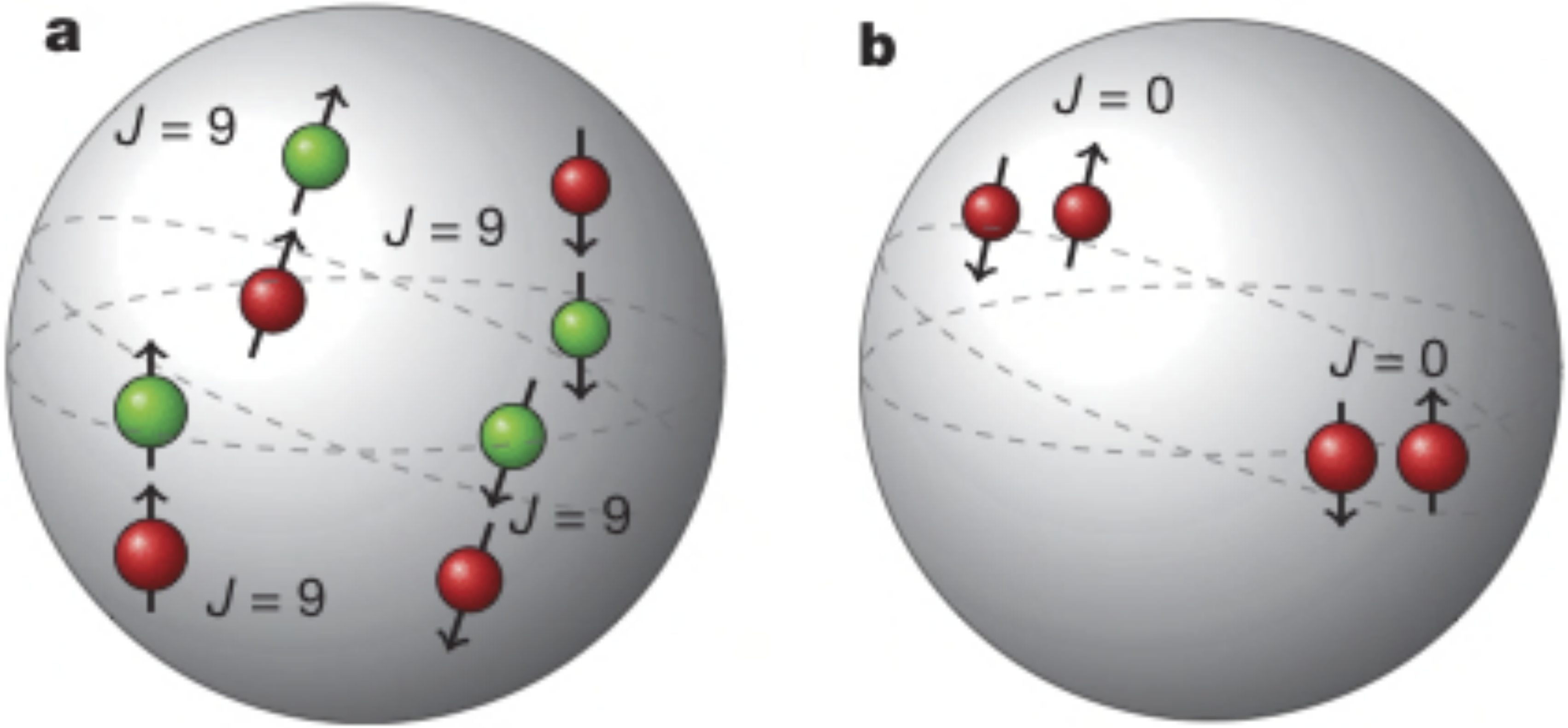
Motivation: A simple description of $N=Z$ nuclei in the $1g_{9/2}$ shell (^{98}In, ^{96}Cd, ^{94}Ag, ^{92}Pd, ^{90}Rh).

Starting point: Shell-model interpretation in terms of spin-aligned T=0 np pairs (Blomqvist).

Experiments have been proposed and carried out at GANIL (Cederwall, de France, Wadsworth...).

What about $N=Z$ nuclei in the $1f_{7/2}$ shell (^{42}Sc, ^{44}Ti, ^{46}V, ^{48}Cr, ^{50}Mn, ^{52}Fe, ^{54}Co)?

Nuclear belly dancer



B. Cederwall *et al.*, Nature **469** (2011) 68

Approximations

Hypothesis: $N=Z$ nuclei can be described in the (spherical) shell model, in an appropriate model space and with an appropriate interaction.

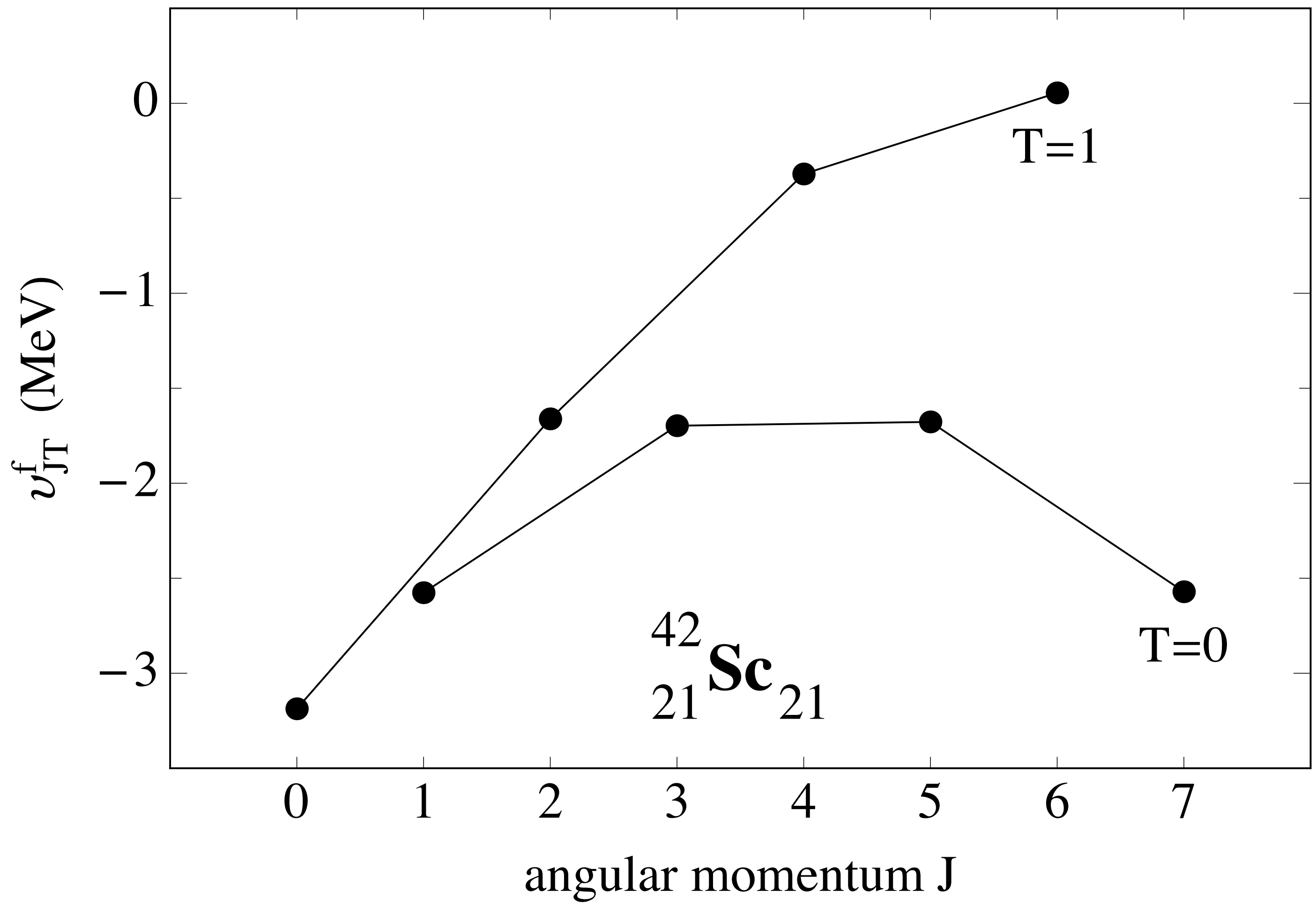
Approximations:

(A) Truncate shell model to a single high-j shell.

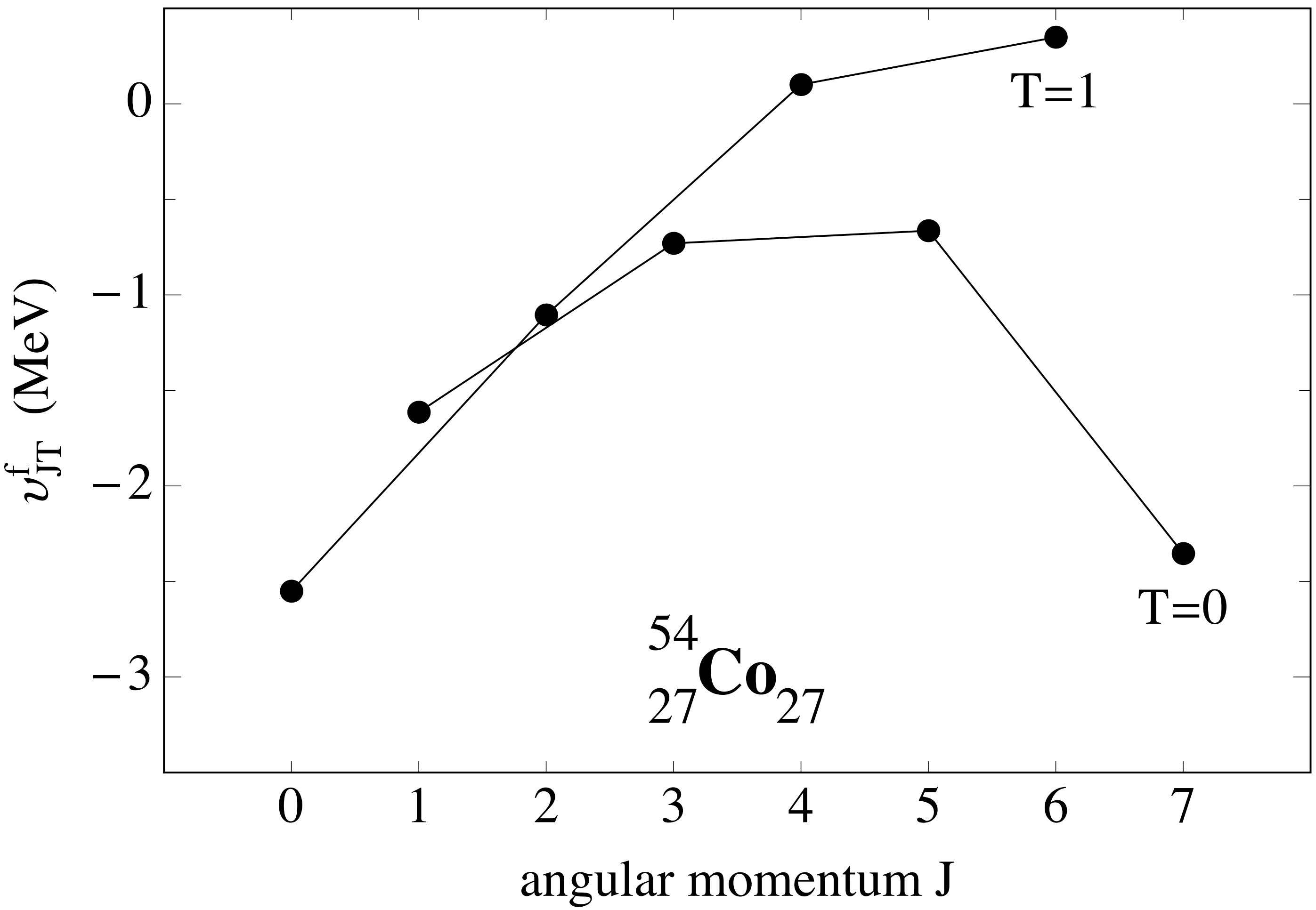
(B) Truncate single-j shell space to one written in terms of aligned-spin B ($J=7$ or 9) pairs.

(C) Replace aligned-spin B pairs by b bosons.

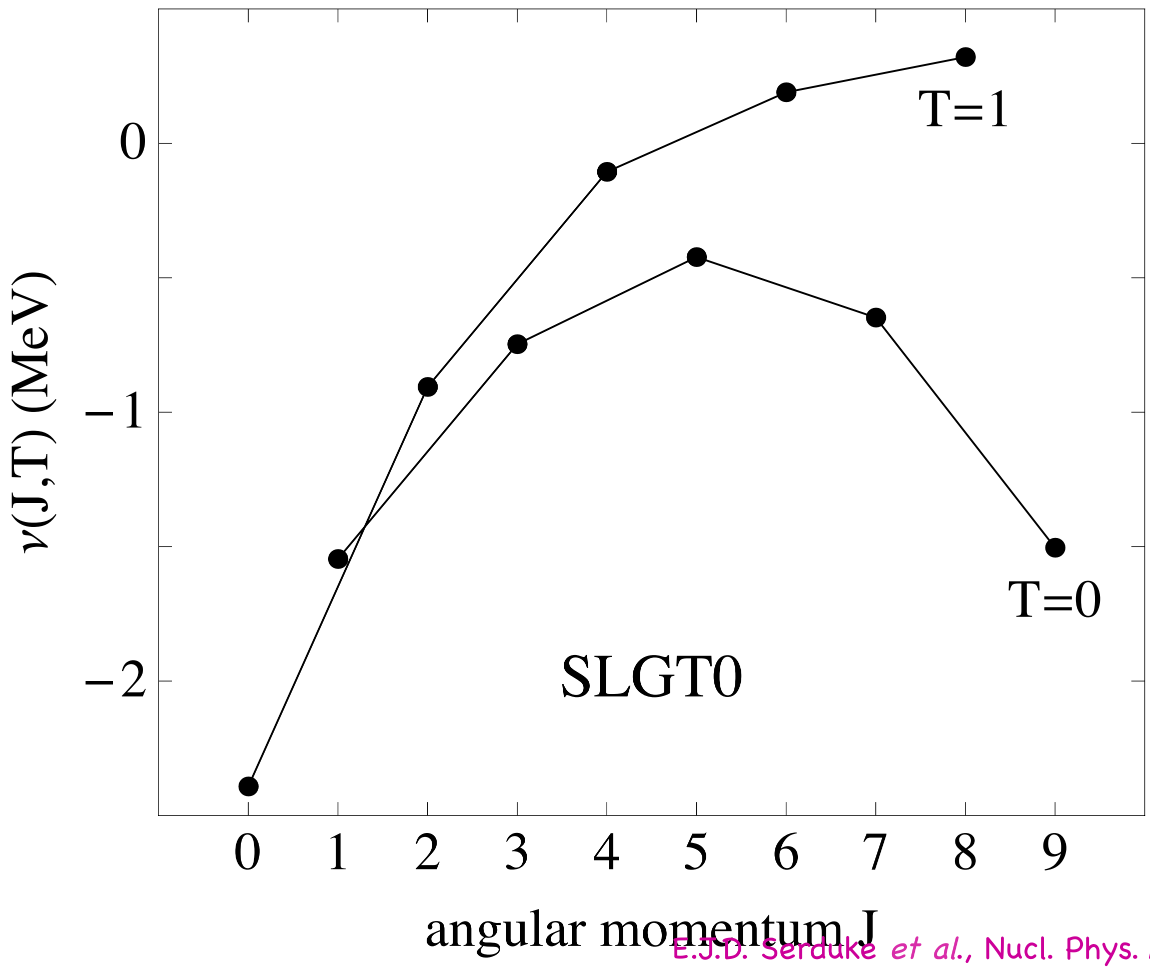
Shell-model interaction: $1f_{7/2}$



Shell-model interaction: $1f_{7/2}$



Shell-model interaction: $1g_{9/2}$



E.J.D. Serduke *et al.*, Nucl. Phys. A **256** (1976) 45
H. Herndl and B.A. Brown, Nucl Phys. A **627** (1997) 35

Pair analysis in the shell model

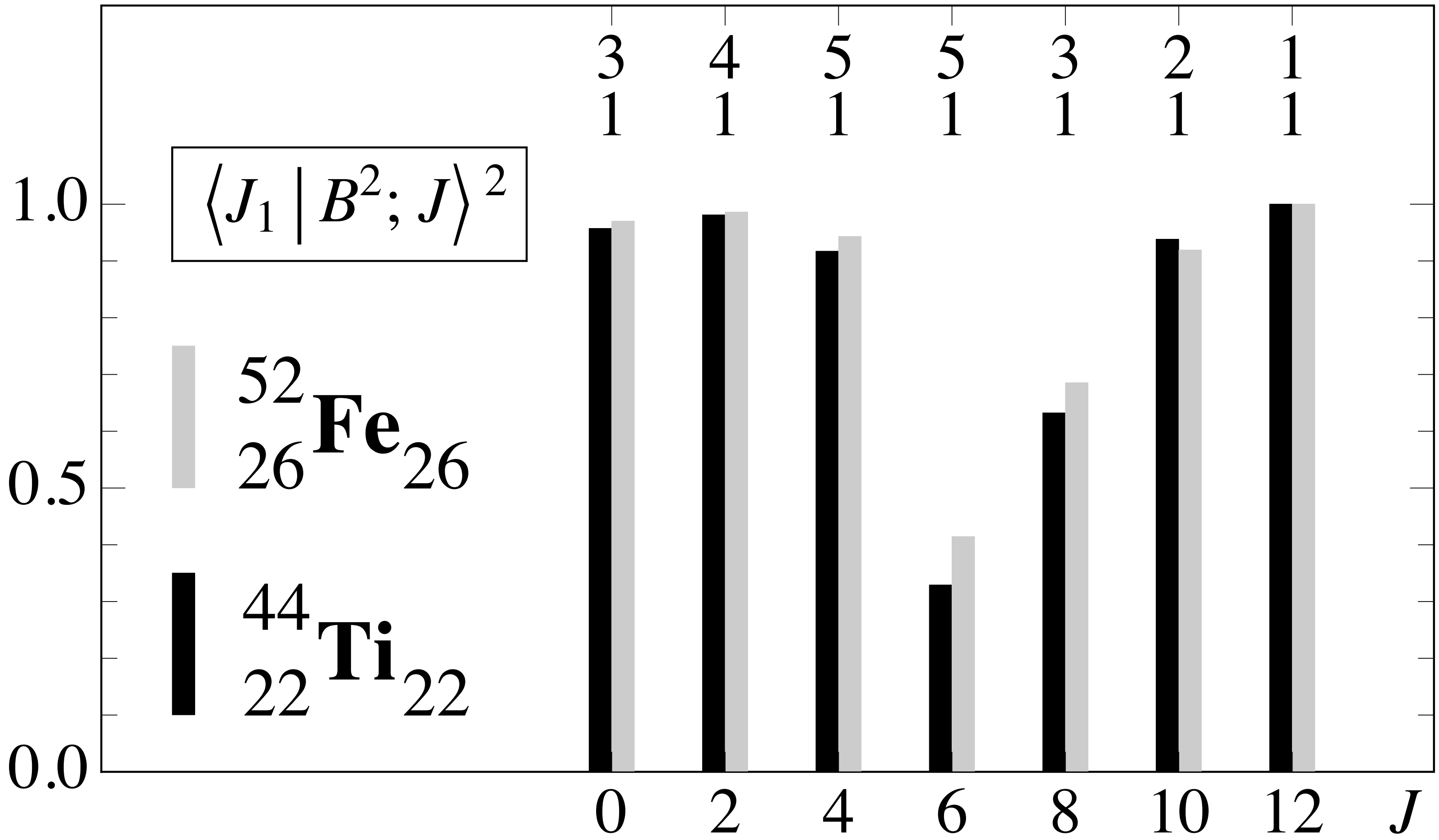
Define different types of nucleon pairs:

$$B_{JT}^{+} = \left(a_{j1/2}^{+} \times a_{j1/2}^{+}\right)^{(JT)}$$

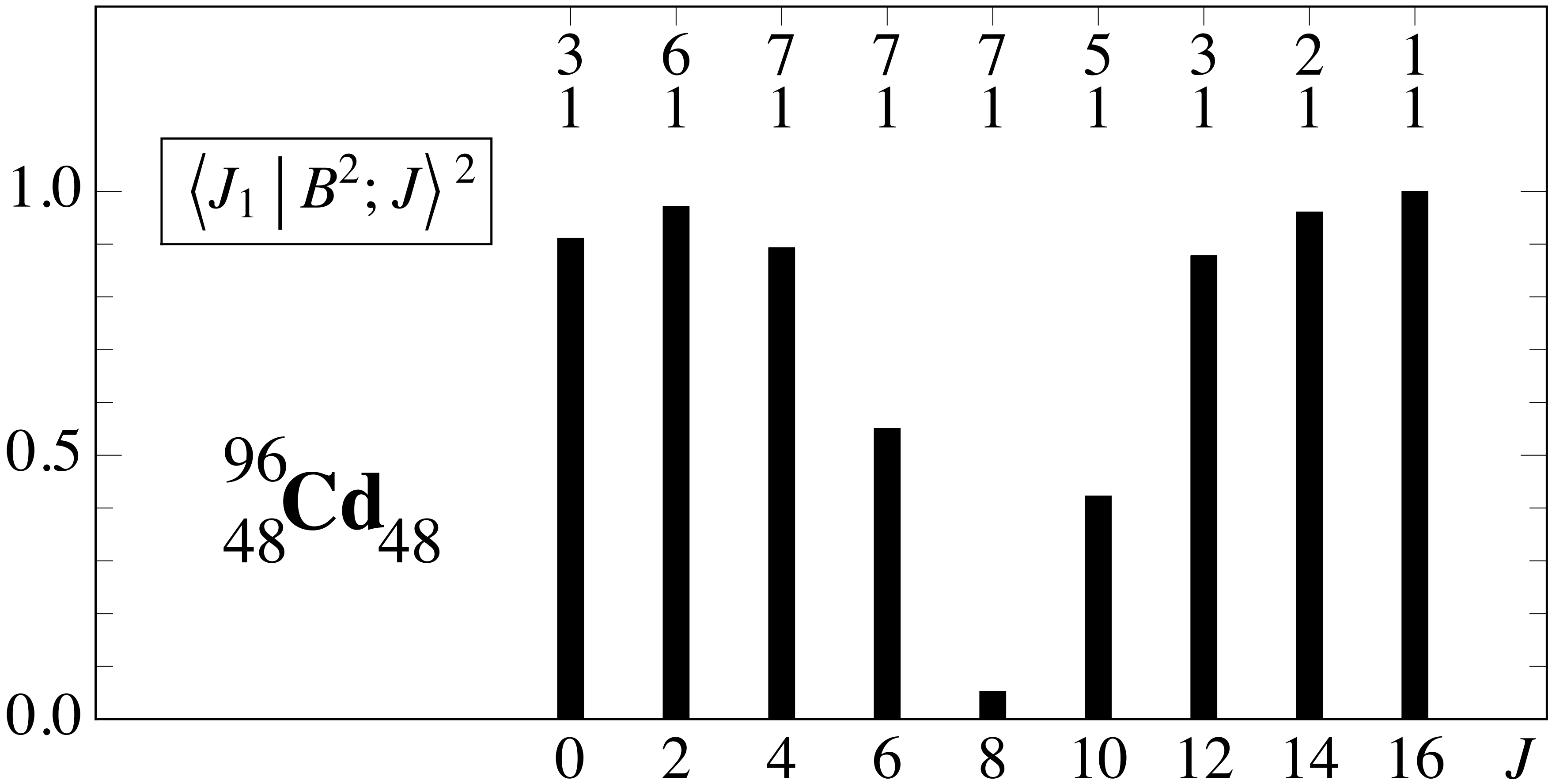
$$S^{+}: J = 0, T = 1; \quad D^{+}: J = 2, T = 1; \quad B^{+}: J = 7 \text{ or } 9, T = 0.$$

Calculate overlap with shell-model wave functions with the nucleon-pair shell model in an isospin-invariant formulation.

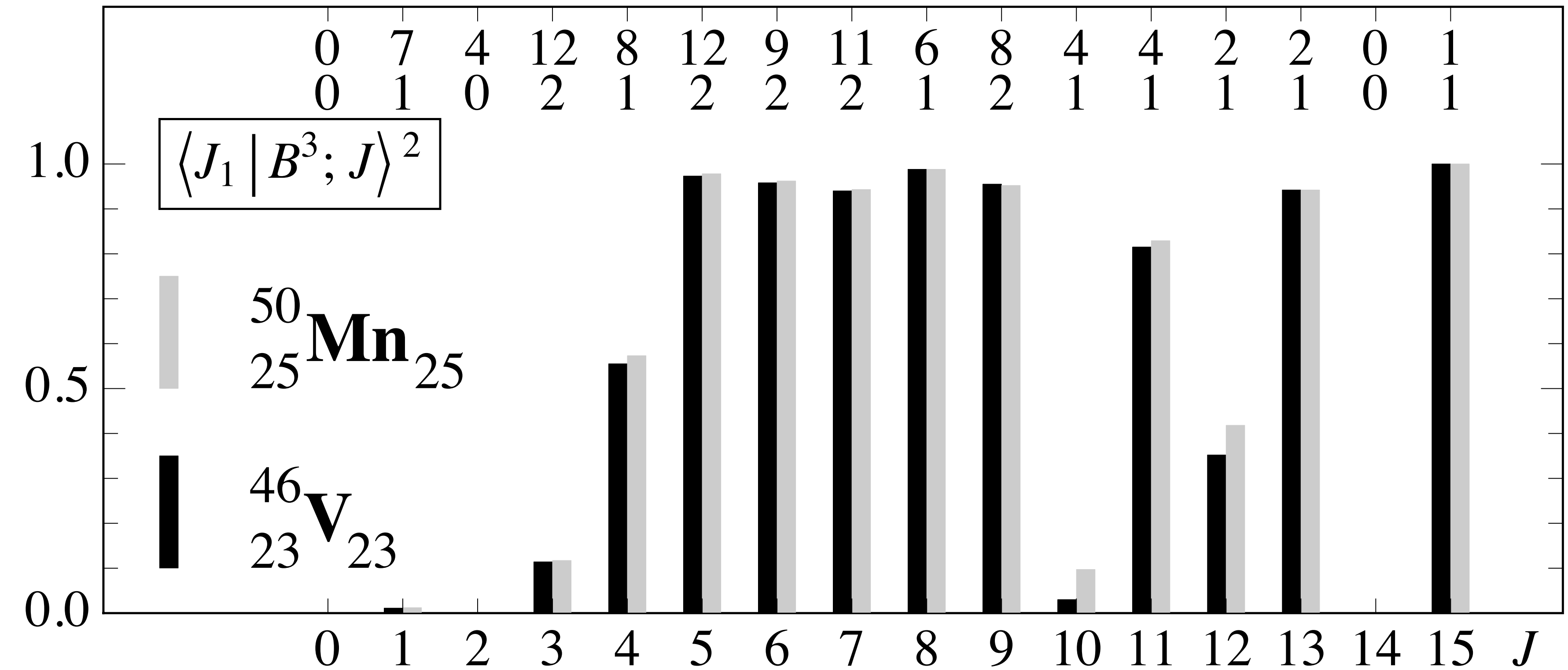
B-pair analysis of ^{44}Ti and ^{52}Fe



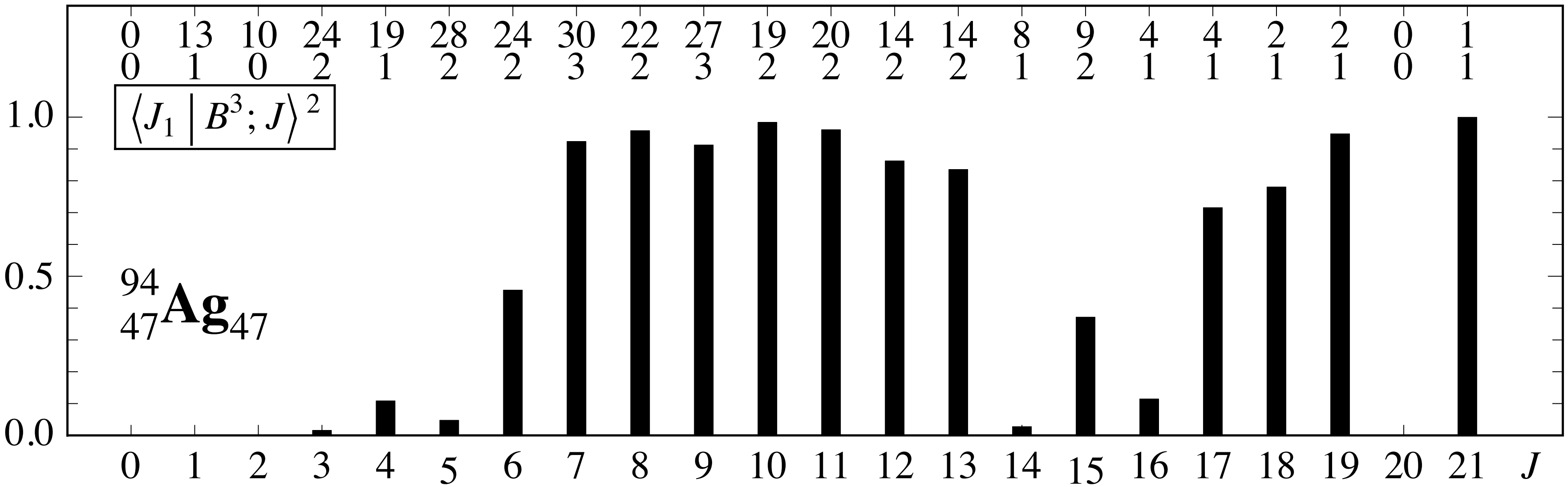
B–pair analysis of ^{96}Cd



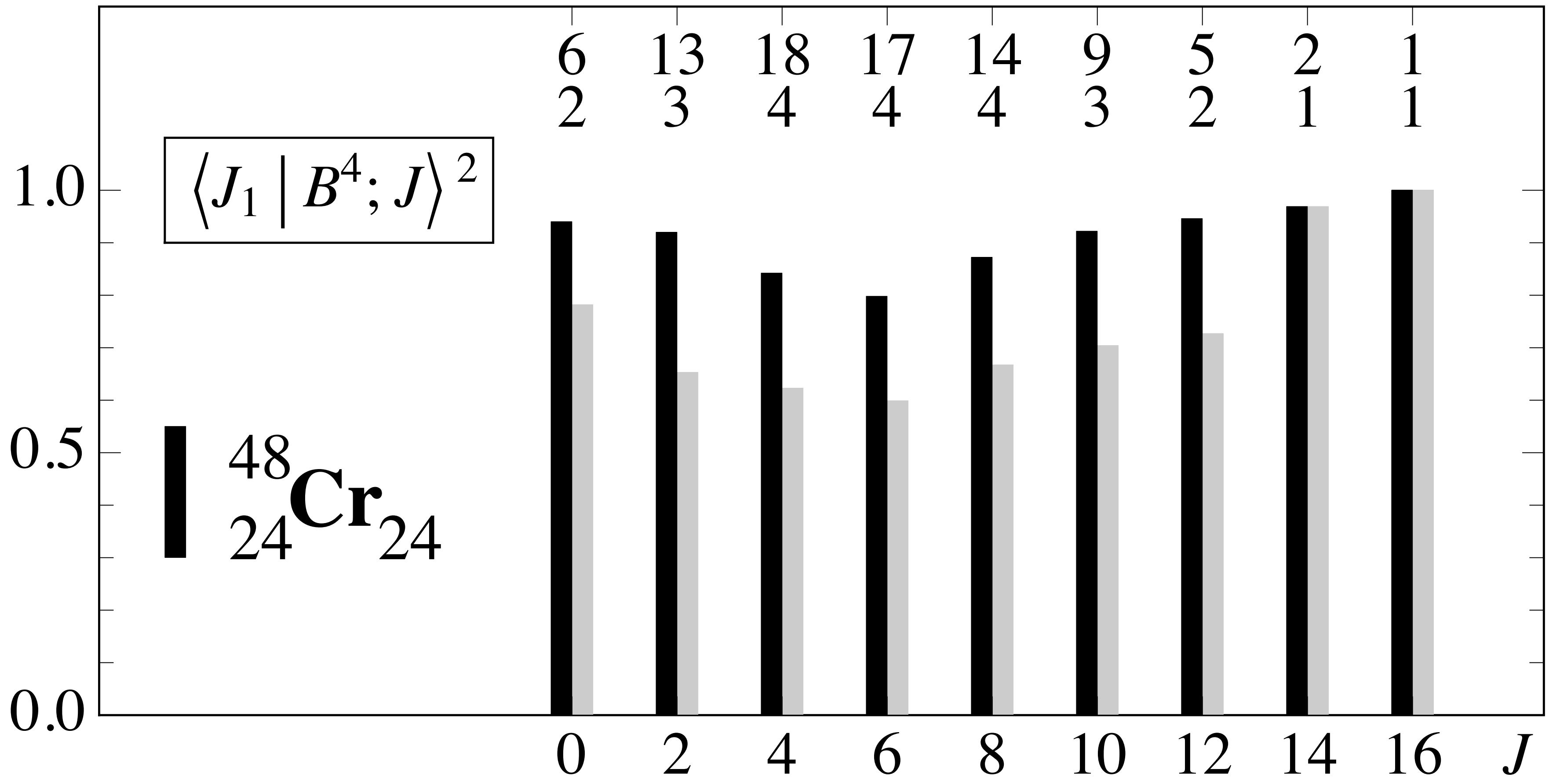
B-pair analysis of ^{46}V and ^{50}Mn



B-pair analysis of ^{94}Ag



B–pair analysis of ^{48}Cr



Mapping to bosons

Transform to a much simpler problem in terms of interacting bosons: $B^+ \to b^+$

Boson energies and boson-boson interactions are derived from the shell model (i.e., *not* fitted).

Magnetic dipole moments

For any state in a single-*j* shell

$$g(\alpha J) = \frac{1}{2}(g_\nu + g_\pi) \approx \begin{cases} 0.52 \text{ to } 0.55\ \mu_\mathrm{N} & (1f_{7/2}) \\ 0.51 \text{ to } 0.54\ \mu_\mathrm{N} & (1g_{9/2}) \end{cases}$$

The same result is obtained with *b*-IBM mapped from a single-*j* shell model.

∴ Magnetic dipole moments test approximation (A) but are insensitive to (B) and (C).

In ^{46}V: $\mu(3_1^+) = 1.64(3)\ \mu_\mathrm{N}$

In ^{50}Mn: $\mu(5_1^+) = 2.76(1)\ \mu_\mathrm{N}$

F.C. Charlwood *et al.*, Phys. Lett. B **690** (2010) 346

Q moment of 5^+ state in ^{50}Mn

Experimental value: $Q\left(5_1^+\right) = 0.80(12)\ \mathrm{b}$

Large-scale shell model: $Q\left(5_1^+\right) = 0.58\ \mathrm{b}$

Numerical result in the $1f_{7/2}$ shell:

$$Q\left(5_1^+\right) = 4.2\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2$$

Expression in terms of b bosons:

$$Q\left(b^3[12]5\right) = \frac{649485}{150241}\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2 \approx 4.3\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2$$

Parameter-free test:

$$\frac{Q\left(5_1^+;{}^{46}\mathrm{V}\right)}{Q\left(7_1^+;{}^{42}\mathrm{Sc}\right)} \approx \frac{Q\left(b^3[12]5\right)}{Q(b)} = \frac{216495}{150241} \approx 1.44$$

F.C. Charlwood *et al.*, Phys. Lett. B **690** (2010) 346

Q moment of 21^+ isomer in ^{94}Ag

In the $1g_{9/2}$ shell:

$$Q\left(21_1^+\right) = \sqrt{\frac{196}{9}}\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2 \approx 0.42\ \mathrm{b}$$

Expression in terms of b bosons:

$$Q\left(b_\infty^3 21\right) = \sqrt{\frac{81949367824}{3489855625}}\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2 \approx 0.44\ \mathrm{b}$$

∴ Measurement of $Q(21^+)$ tests (A). Calculation confirms (B+C).

Q moment of 7^+ isomer in ^{94}Ag

In the $1g_{9/2}$ shell (84 components for J=7):

$$Q\left(7_1^+\right) = 6.60\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2 \approx 0.60\ \mathrm{b}$$

Expression in terms of b bosons:

$$Q\left(b^3[16]7\right) \approx \sqrt{\frac{3093027730092364}{627253477610841}}\left(e_\nu + e_\pi\right)\left(\ell_{\mathrm{ho}}\right)^2 \approx 0.64\ \mathrm{b}$$

Conclusions

(A) Truncation of the shell model to a single-j shell. (?)

(B) Truncation of the single-j shell space to one written in terms of aligned-spin B (J=7 or 9) pairs. (✓)

(C) Replacement of aligned-spin B pairs by b bosons. (✔)

P. Van Isacker, Int. J. Mod. Phys. E **22** (2013) 1330028

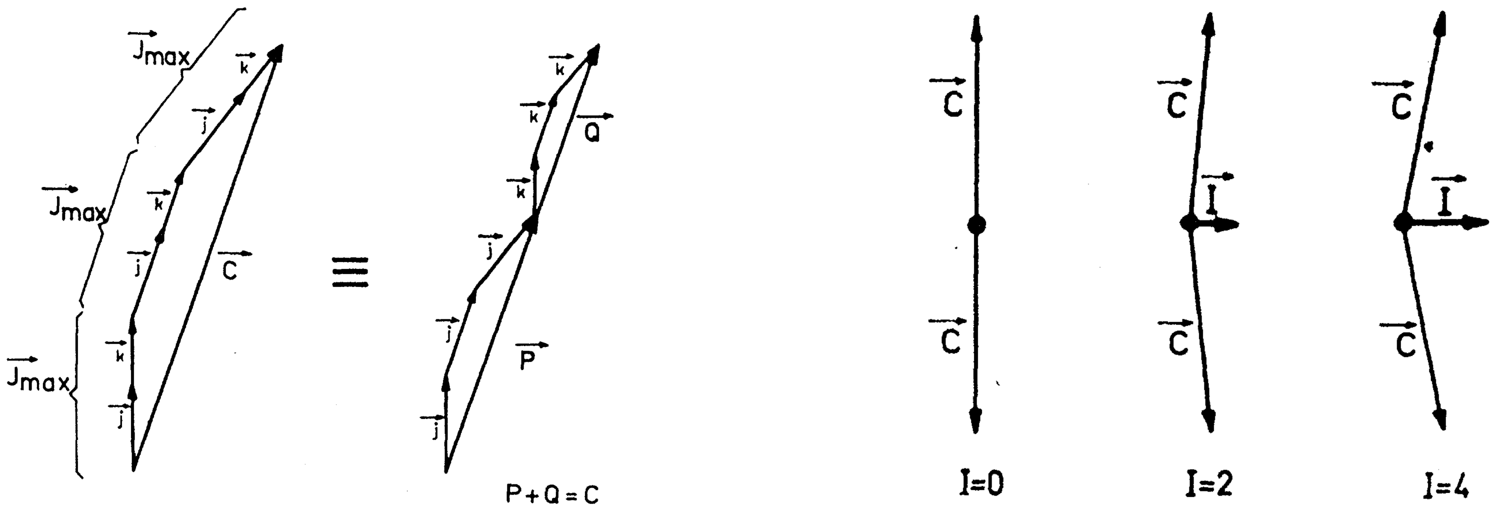
Plus ça change (1)

PHYSICAL REVIEW VOLUME 161, NUMBER 4 20 SEPTEMBER 1967

Stretch Scheme, a Shell-Model Description of Deformed Nuclei

MICHAEL DANOS AND VINCENT GILLET
Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvett, Seine et Oise, France
and
National Bureau of Standards, Washington, D. C.
(Received 23 March 1967)

A good angular-momentum wave function containing the maximum possible intrinsic angular momenta leads to a microscopic description of the nuclear rotational spectra in terms of spherical shell-model states. The rotational excitation energies arise from the residual two-body force. In the actual model calculations, the only approximation was a partial violation of the exclusion principle. The computed departures from the $I(I+1)$ law are consistent with experiment. Reasons are given for the preference of positive over negative intrinsic deformations.



Plus ça change (2)

DL/NUC/P265T

preprint Daresbury Laboratory

DL/NUC/P265T

HIGH MULTIPOLE PROTON-NEUTRON PAIRING IN NUCLEI

by

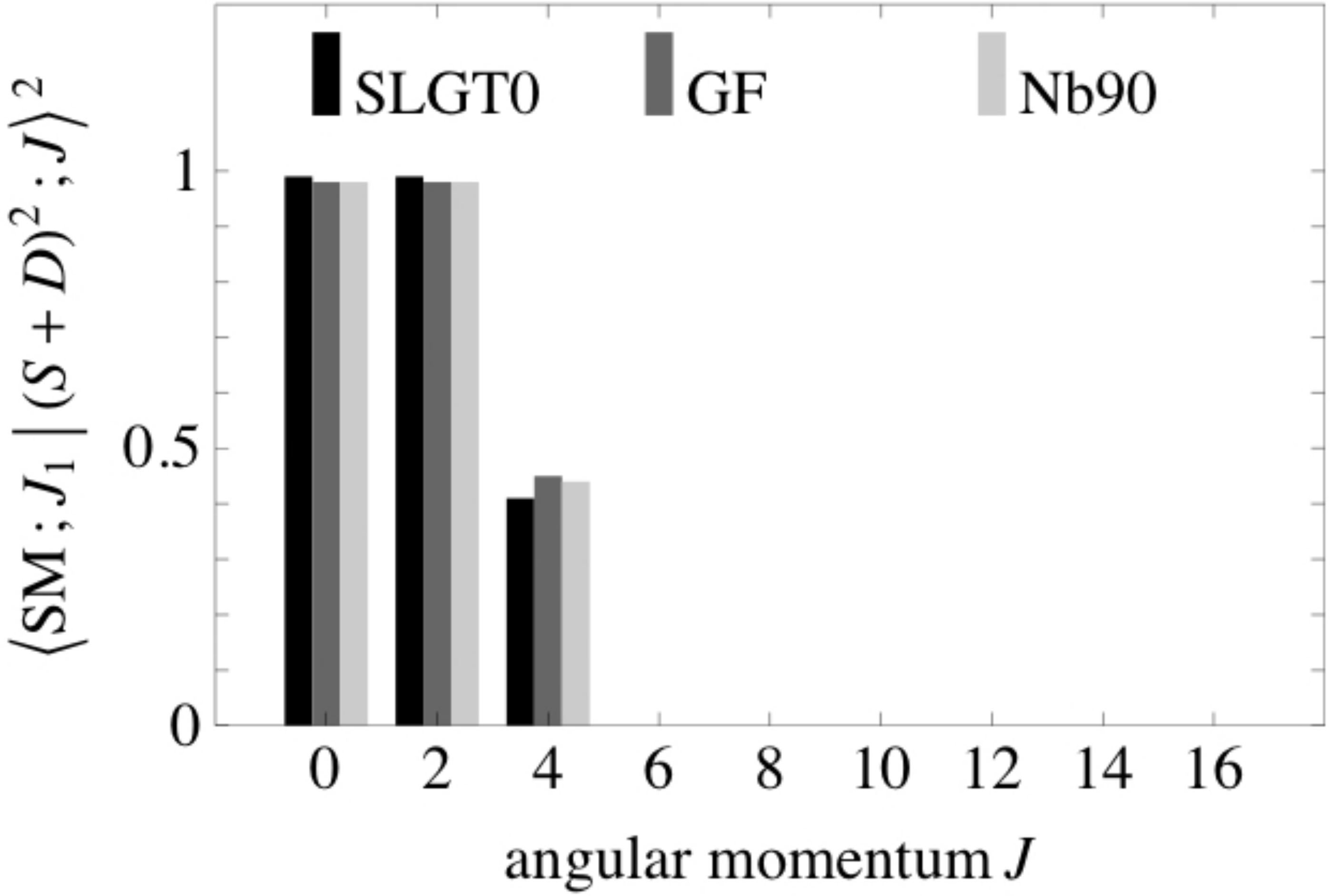
H.J. DALEY, SERC Daresbury Laboratory

To be submitted to Nucl. Phys. A

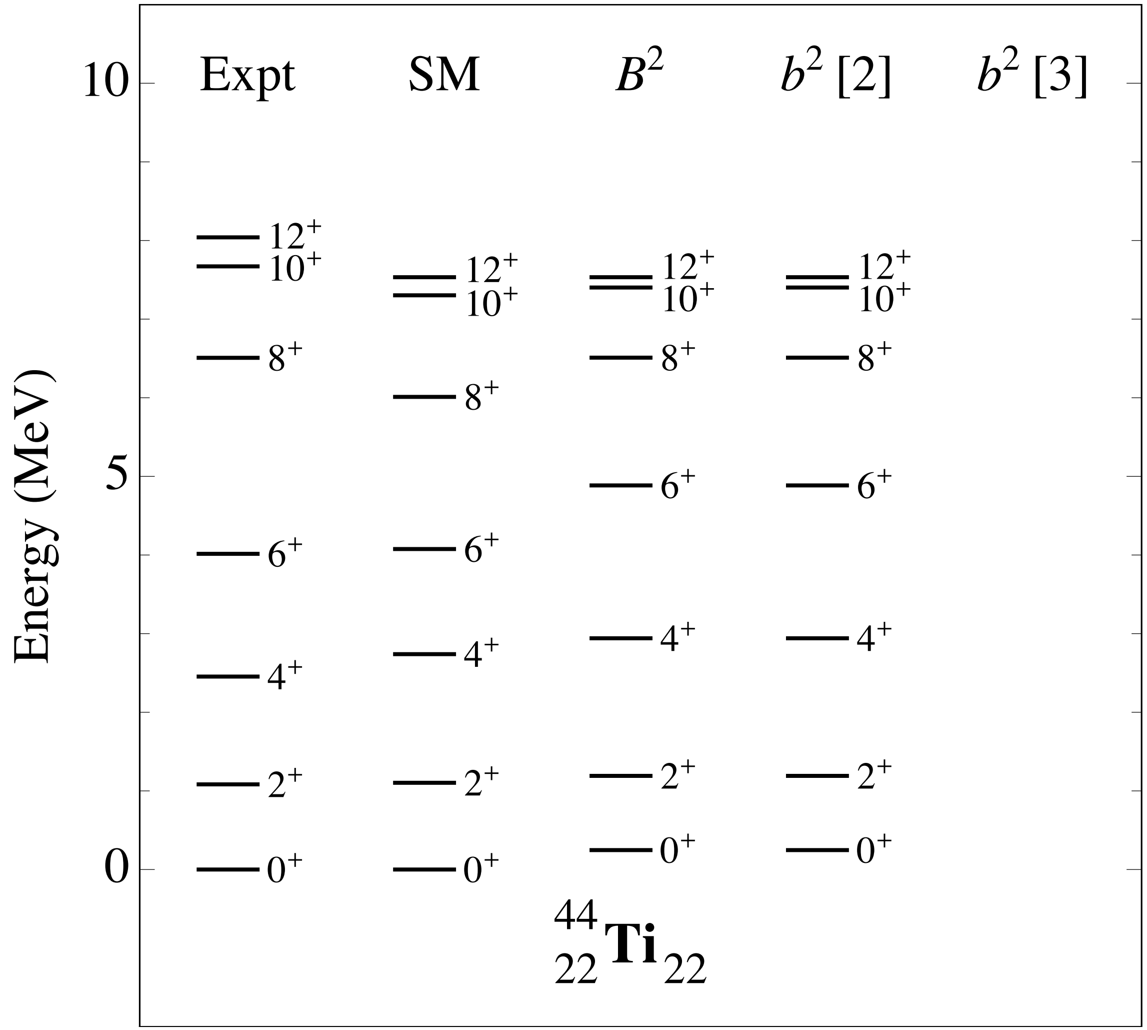
NOVEMBER, 1987

Science and Engineering Research Council
DARESBURY LABORATORY
Daresbury, Warrington WA4 4AD

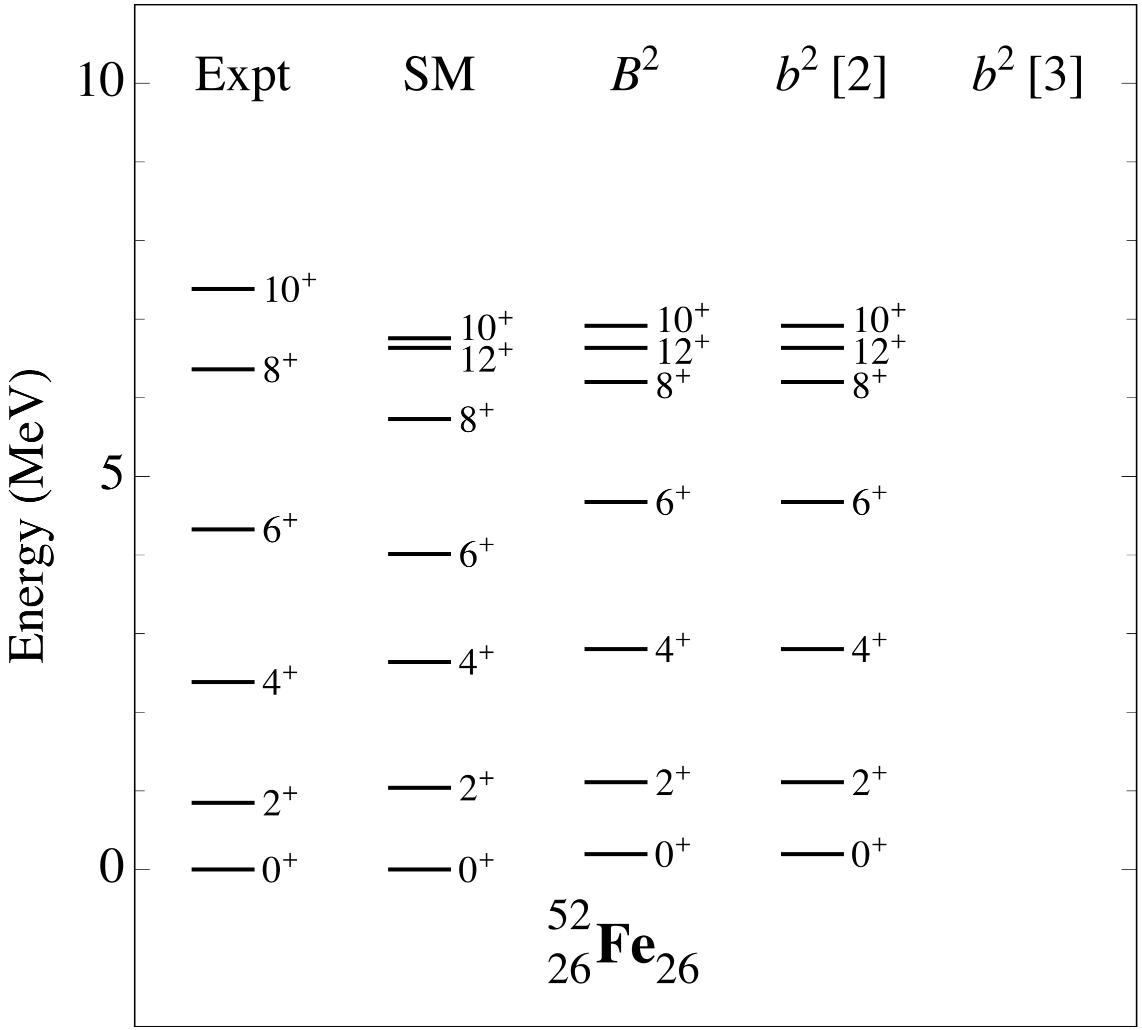
Structure of ^{96}Cd



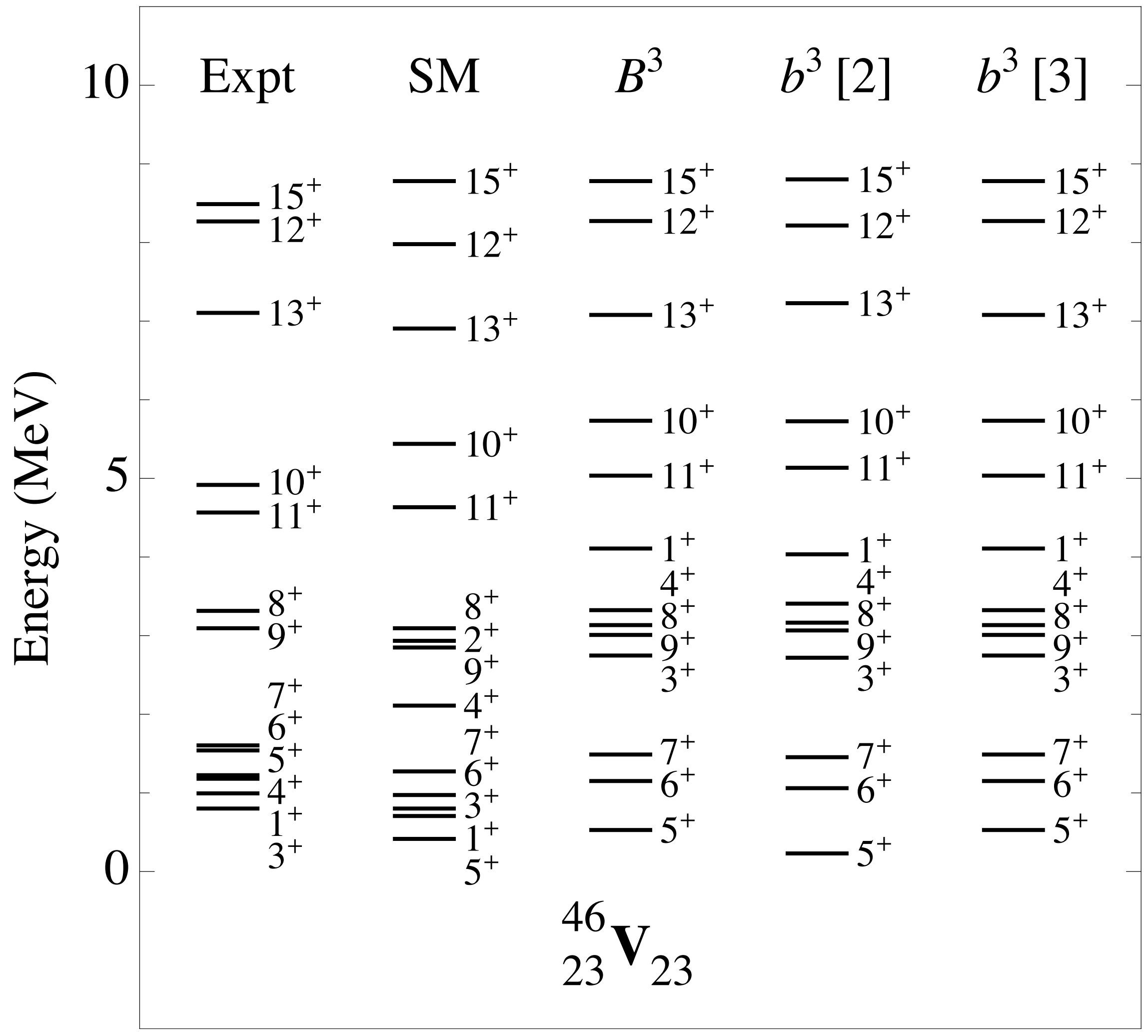
Spectrum of ^{44}Ti



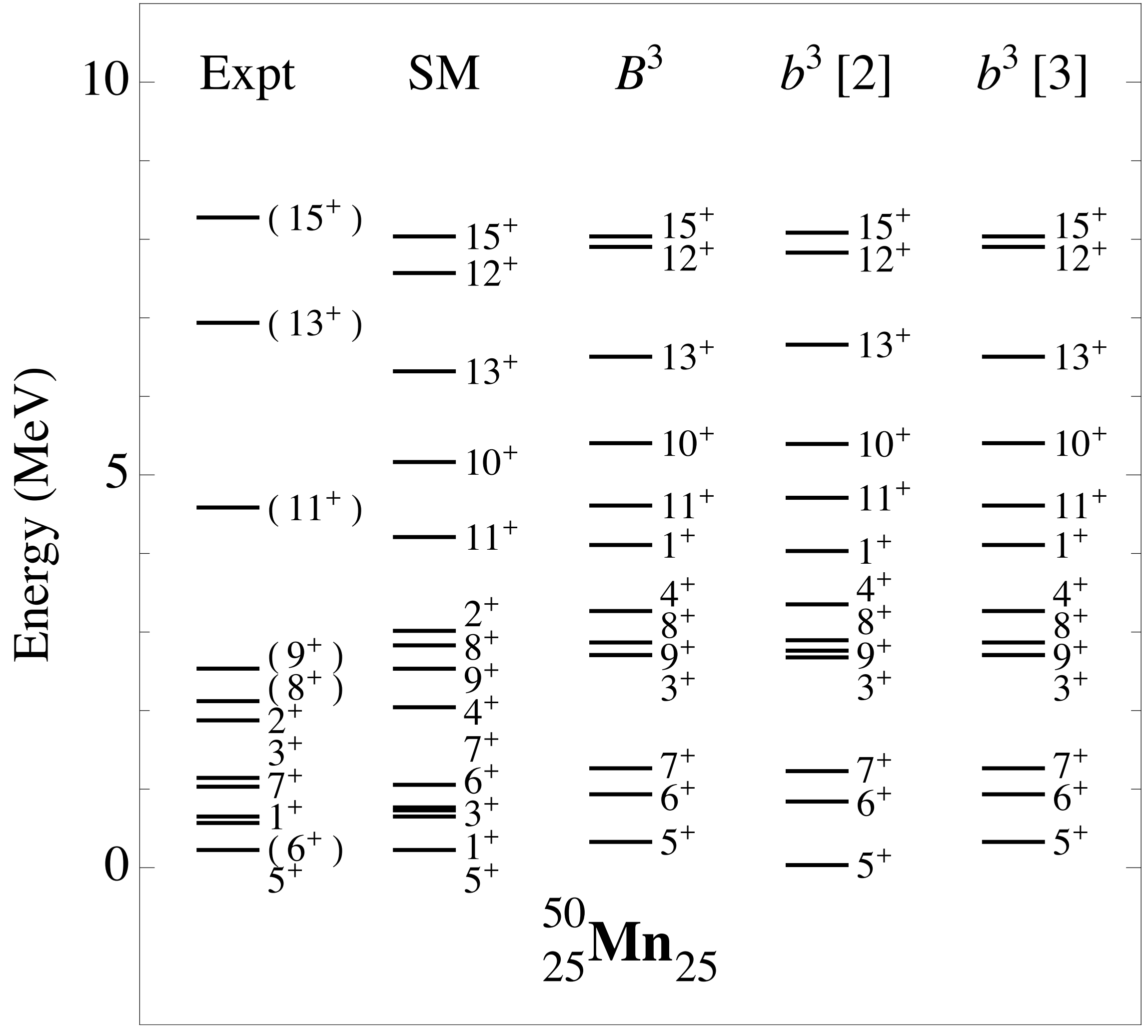
Spectrum of ^{52}Fe



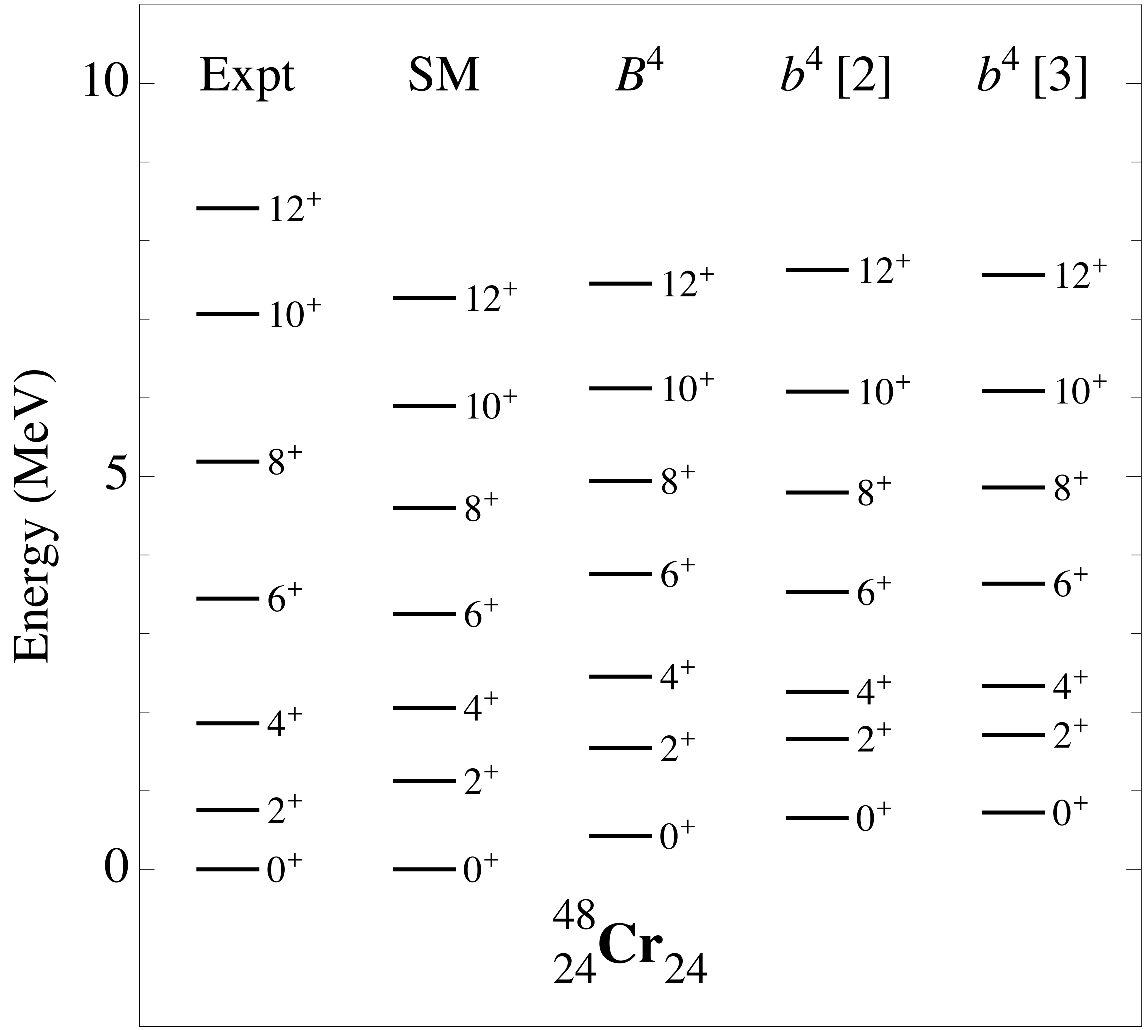
Spectrum of ^{46}V



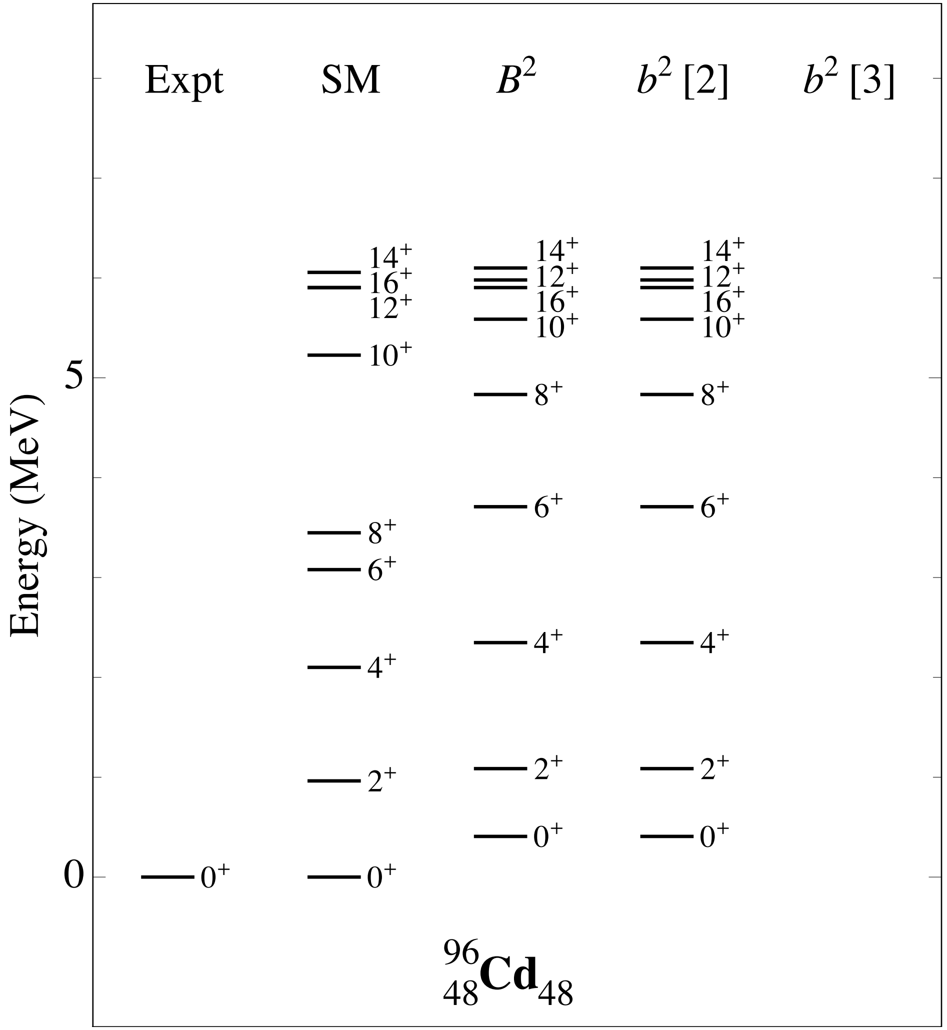
Spectrum of ^{50}Mn



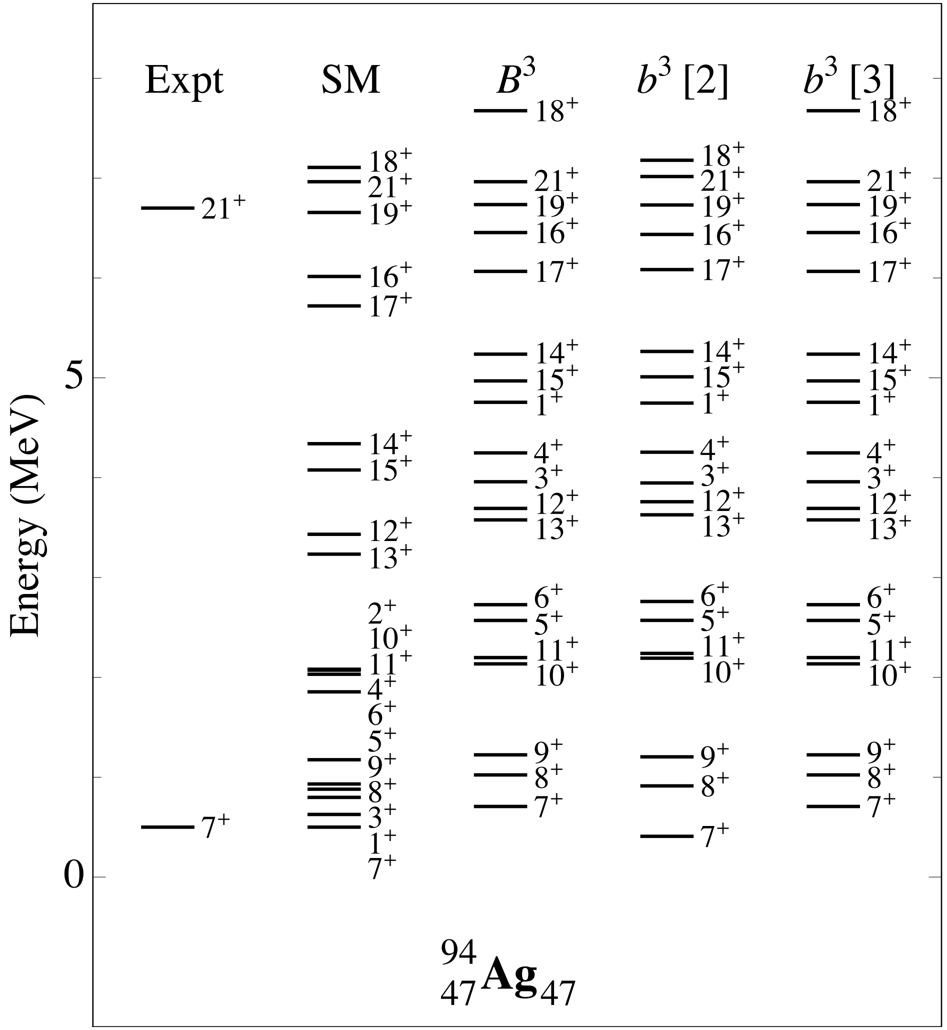
Spectrum of ^{48}Cr



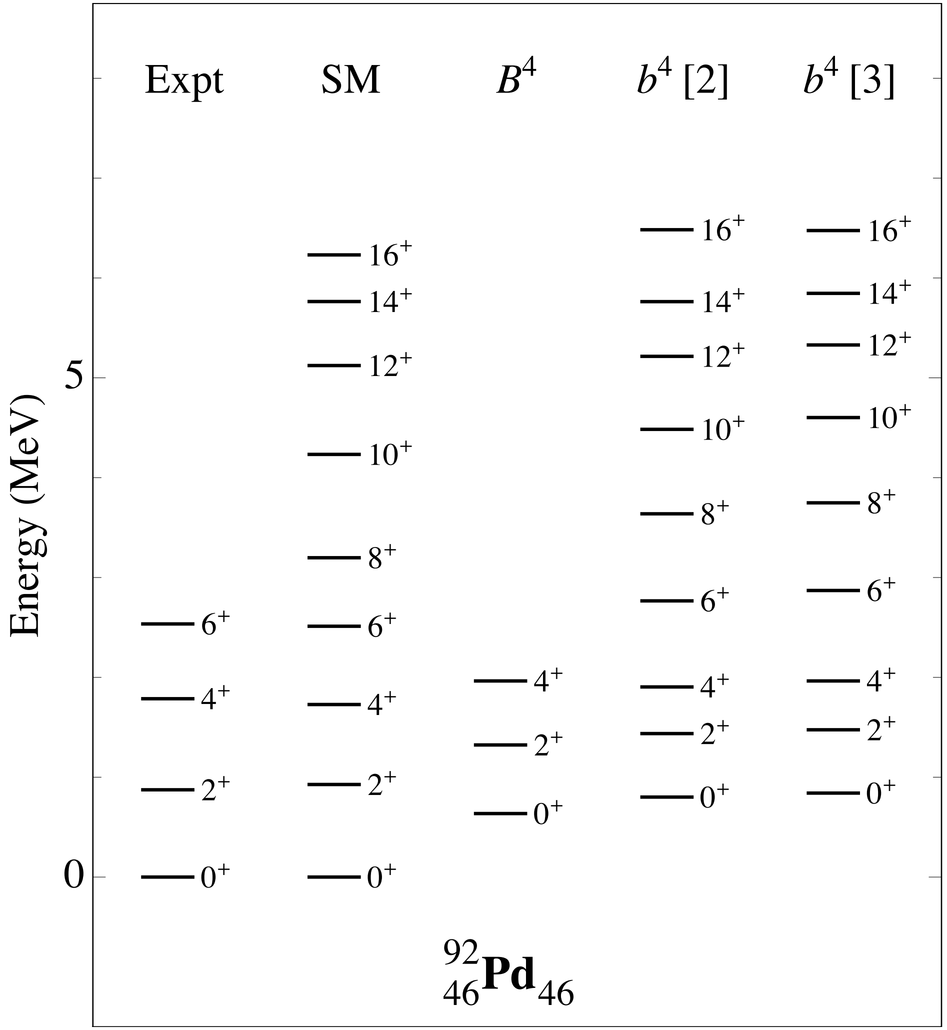
Spectrum of ^{96}Cd



Spectrum of ^{94}Ag



Spectrum of ^{92}Pd



Brix Day, ULB, Brussels, 2 December 2013

E2 properties in *b*-IBM

The E2 operator in the shell model:

$$\hat{T}^{\mathrm{F}}_{\mu}(\mathrm{E2}) = e_{\nu} \sum_{i\in\nu} r_i^2 Y_{2\mu}(\theta_i,\phi_i) + e_{\pi} \sum_{i\in\pi} r_i^2 Y_{2\mu}(\theta_i,\phi_i)$$

The E2 operator in terms of *b* bosons:

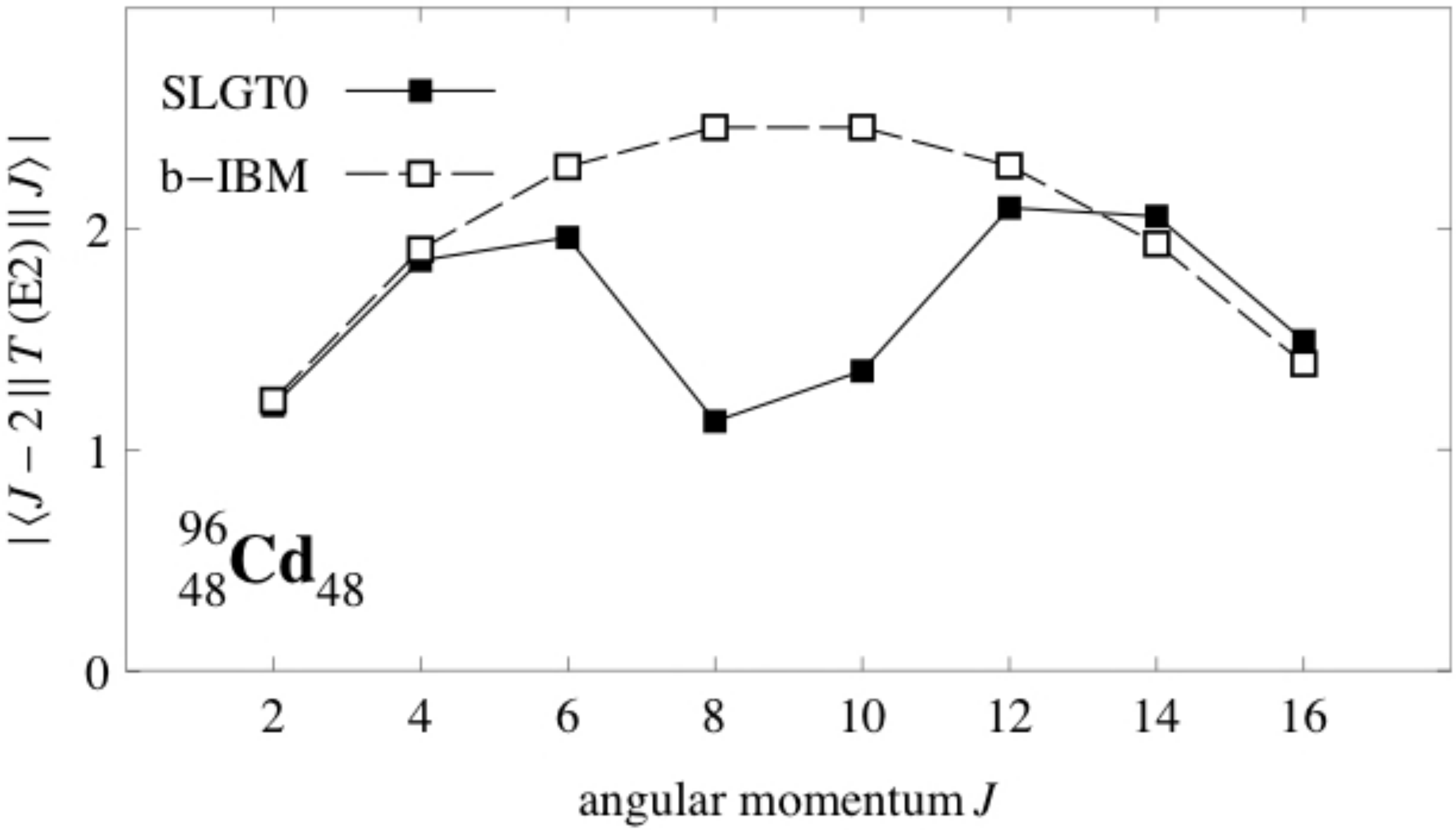
$$\hat{T}^{\mathrm{B}}_{\mu}(\mathrm{E2}) = e_{\mathrm{b}} \left(b^{+} \times \tilde{b}\right)^{(2)}_{\mu}$$

The mapping implies the relation:

$$\left\langle (g_{9/2})^2;9^+ \left\| \hat{T}^{\mathrm{F}}(\mathrm{E2}) \right\| (g_{9/2})^2;9^+ \right\rangle = \langle b \| \hat{T}^{\mathrm{B}}(\mathrm{E2}) \| b \rangle$$

$$\Rightarrow e_{\mathrm{b}} = \sqrt{\frac{55}{3\pi}} (\ell_{\mathrm{ho}})^2 \times \sqrt{\frac{266}{187}} (e_{\nu} + e_{\pi})$$

B(E2) values in ^{96}Cd

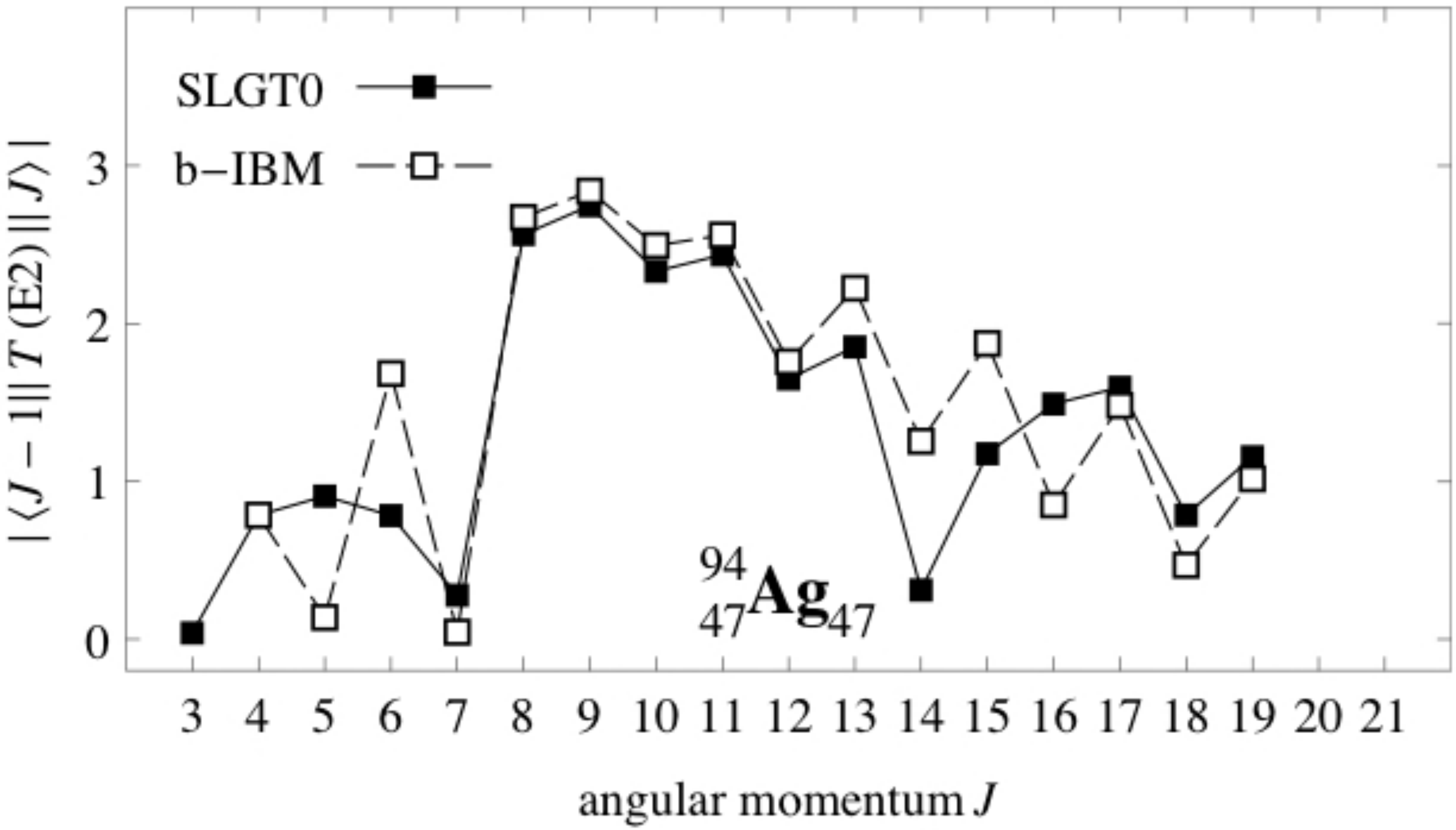


B(E2) values in ^{96}Cd

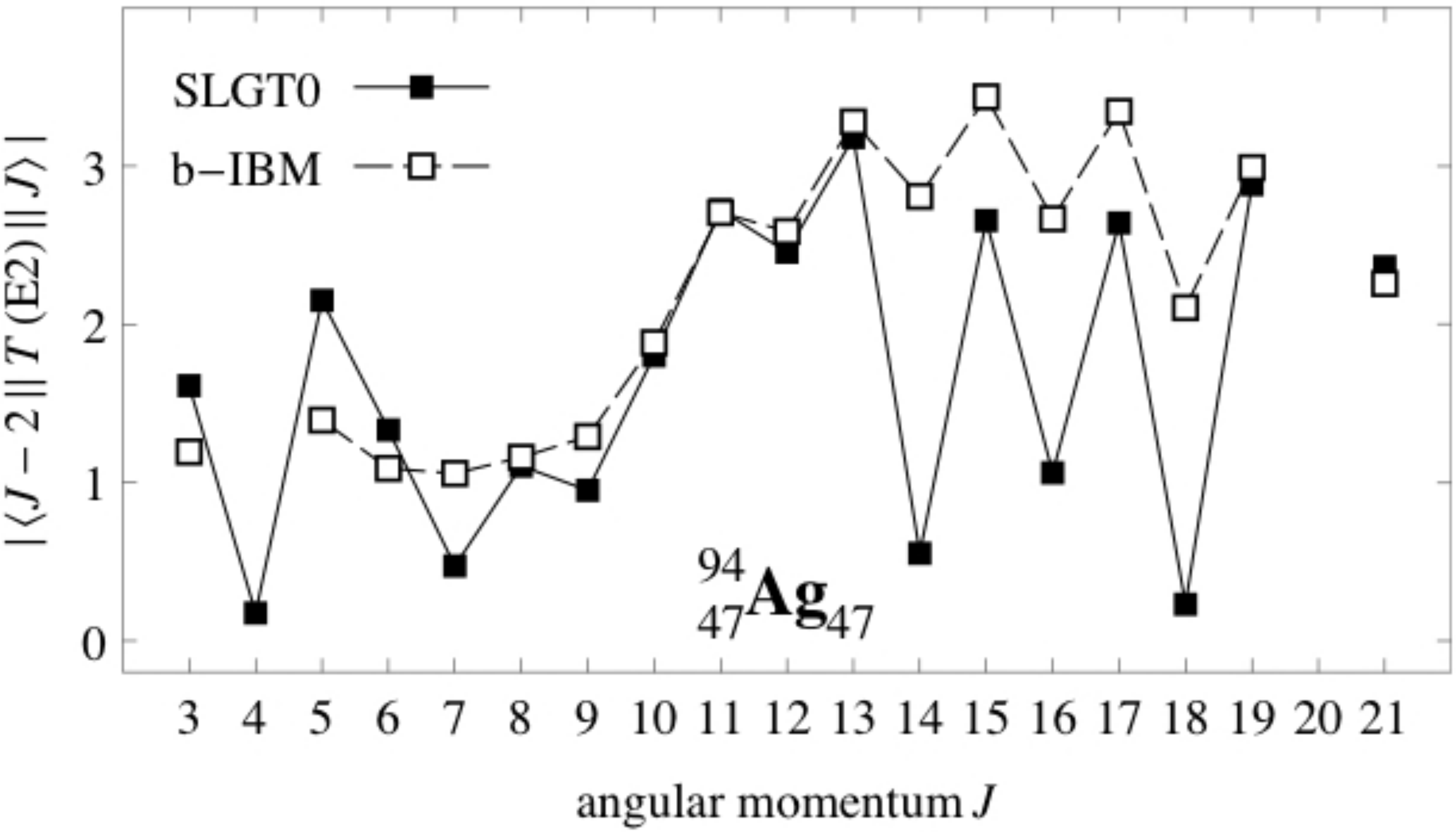
A simple consequence of the aligned-pair assumption:

$$B(\mathrm{E2}; J+2 \to J) = e_{\mathrm{b}}^2 20(2J+1) \begin{Bmatrix} 9 & 9 & 2 \\ J+2 & J & 9 \end{Bmatrix}^2$$

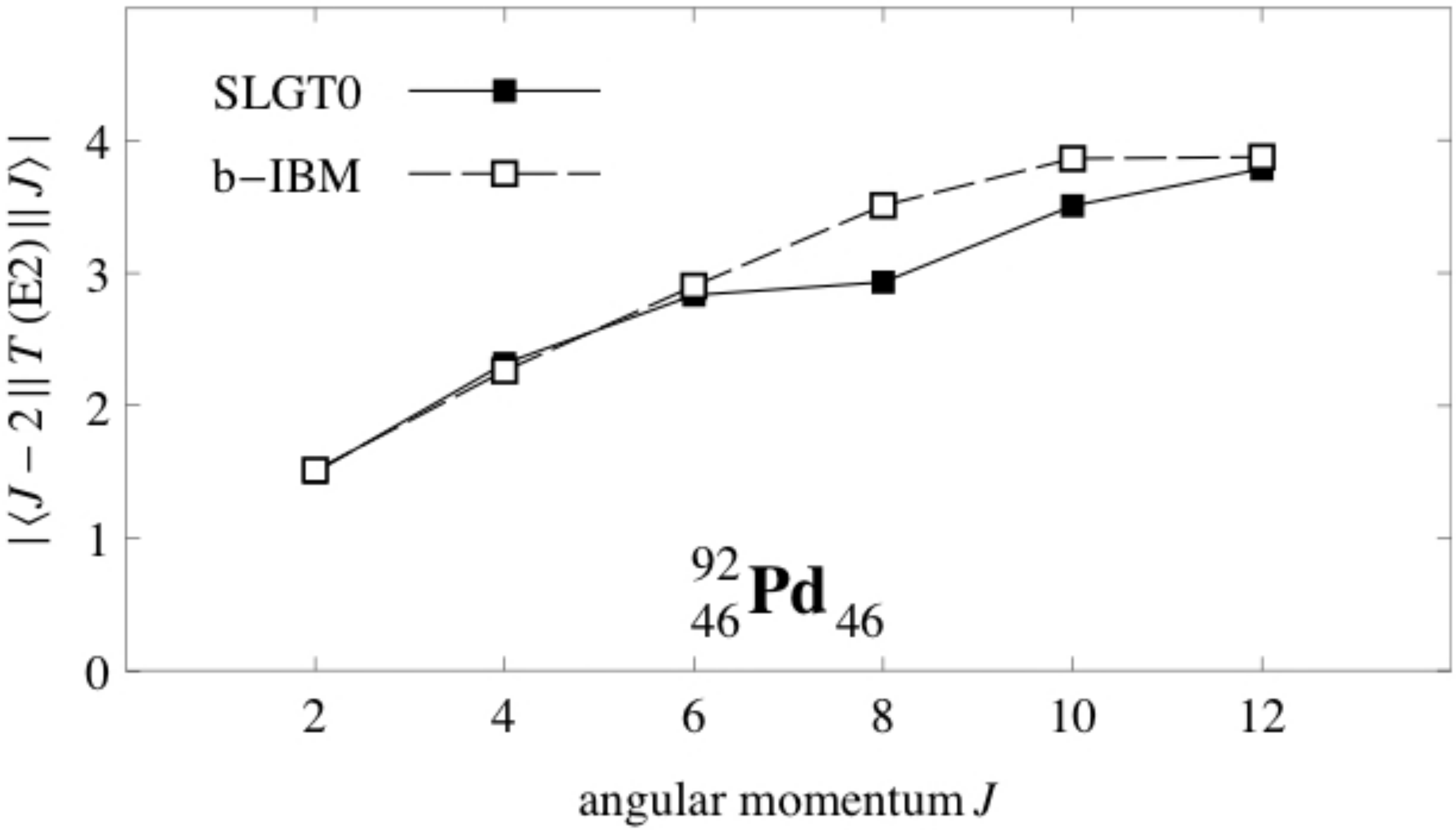
B(E2) values in ^{94}Ag



B(E2) values in ^{94}Ag



B(E2) values in ^{92}Pd



Energy of 21+ isomer in ^{94}Ag

How many independent states of three *b*-bosons can couple to angular momentum 21?

$$d(\upsilon,\ell,J)=\frac{i}{2\pi}\oint_{|z|=1}\frac{\left(z^{2J+1}-1\right)\left(z^{2\upsilon+2\ell-1}-1\right)\prod_{k=1}^{2\ell-2}\left(z^{\upsilon+k}-1\right)}{z^{\ell\upsilon+J+2}\prod_{k=1}^{2\ell-2}\left(z^{k+1}-1\right)}$$

Answer: *d* (3,9,21)=2 one of which is spurious.

After elimination of the spurious state, the energy of the physical 21+ state is

$$E\left(21^{+}\right)=3\varepsilon_{b}+\frac{6851}{20155}\nu_{12}^{\mathrm{b}}+\frac{15488}{21545}\nu_{14}^{\mathrm{b}}+\frac{1212882}{624805}\nu_{16}^{\mathrm{b}}$$

P. Van Isacker, Phys. Scr. **T150** (2012) 014042

Energy of 21^+ isomer in ^{94}Ag

In turn, we know the boson matrix elements in terms of the shell-model matrix elements:

$$\nu_{12}^{b} = \frac{1218}{69355}\nu_3 + \frac{63423}{138710}\nu_4 + \frac{29957}{63050}\nu_5 + \frac{109881}{53350}\nu_6 + \frac{1148337}{2358070}\nu_7 + \frac{15231}{31525}\nu_8 + \frac{10893}{535925}\nu_9$$

$$\nu_{14}^{b} = \frac{868}{8515}\nu_5 + \frac{1953}{1310}\nu_6 + \frac{46251}{57902}\nu_7 + \frac{1977}{1310}\nu_8 + \frac{2211}{22270}\nu_9$$

$$\nu_{16}^{b} = \frac{8}{17}\nu_7 + 3\nu_8 + \frac{9}{17}\nu_9$$

Energy of 21^+ isomer in ^{94}Ag

Therefore, we know the energy of the 21^+ isomer in ^{94}Ag in terms of the shell-model interaction matrix elements:

$$E_\mathrm{b}\left(21^+\right) = \frac{22134}{3707825}\,\nu_3 + \frac{1152549}{7415650}\,\nu_4 + \frac{1347751953}{5740387250}\,\nu_5 + \frac{8606149749}{4857250750}\,\nu_6 + \frac{354940047213}{214690483150}\,\nu_7 + \frac{1561553973}{220784125}\,\nu_8 + \frac{15411107094}{3753330125}\,\nu_9$$

Energy of 21^+ isomer in ^{94}Ag

The 21^+ state is unique in the $1g_{9/2}$ shell model.
Its energy is therefore known analytically:

$$E_\mathrm{f}\left(21^+\right) = \frac{21}{65}\ \nu_5 + \frac{21}{10}\ \nu_6 + \frac{645}{442}\ \nu_7 + \frac{69}{10}\ \nu_8 + \frac{717}{170}\ \nu_9$$

Comparison tests the reliability of the mapping:

$$E_\mathrm{f}\left(21^+\right) \approx 0.323\ \nu_5 + 2.1\ \nu_6 + 1.459\ \nu_7 + 6.9\ \nu_8 + 4.218\ \nu_9$$

$$E_\mathrm{b}\left(21^+\right) \approx 0.006\ \nu_3 + 0.155\ \nu_4 + 0.235\ \nu_5 + 1.772\ \nu_6 + 1.653\ \nu_7 + 7.073\ \nu_8 + 4.106\ \nu_9$$

Conservation of *n*, *J* and *T*

A unique *n*-particle shell-model state with angular momentum *J* and isospin *T* has energy

$$E_{\mathrm{f}}\left(j^n JT\right) = \sum_{\lambda} a_\lambda v_\lambda$$

The coefficients a_λ satisfy

$$\sum_{\lambda=0}^{2j} a_\lambda = \frac{n(n-1)}{2},$$

$$\sum_{\lambda=0}^{2j} \lambda(\lambda+1) a_\lambda = J(J+1) + j(j+1) \times n(n-2)$$

$$\sum_{\substack{\lambda=0\\ \text{even}}}^{2j} 2a_\lambda = T(T+1) + \frac{3}{4} n(n-2)$$

Pitfalls of non-orthogonal bases

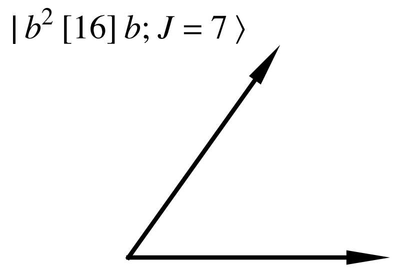
The 7^+ isomer in ^{94}Ag can be written differently:

$$\left|7^+\right\rangle \approx \left|b^2[4]\,b;J=7\right\rangle$$

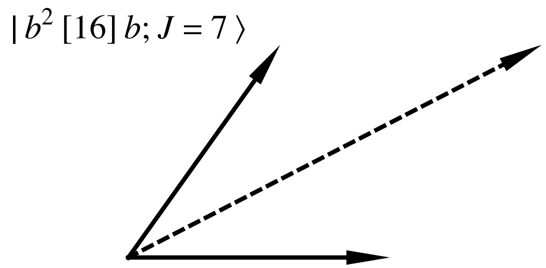
Because

$$\left\langle b^2[4]\,b;J=7\middle|b^2[16]\,b;J=7\right\rangle = \sqrt{\frac{7012200}{8733503}} = 0.896$$

Pitfalls of non-orthogonal bases



Pitfalls of non-orthogonal bases



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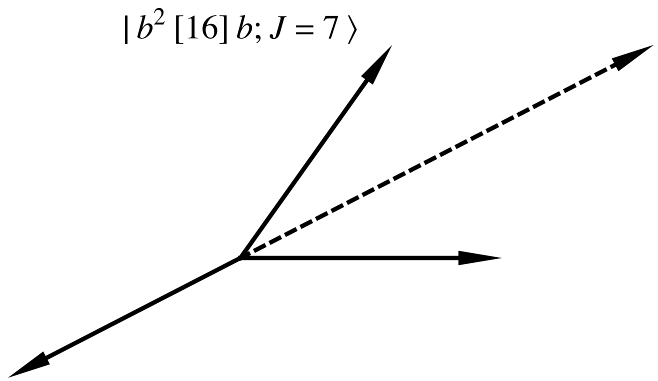

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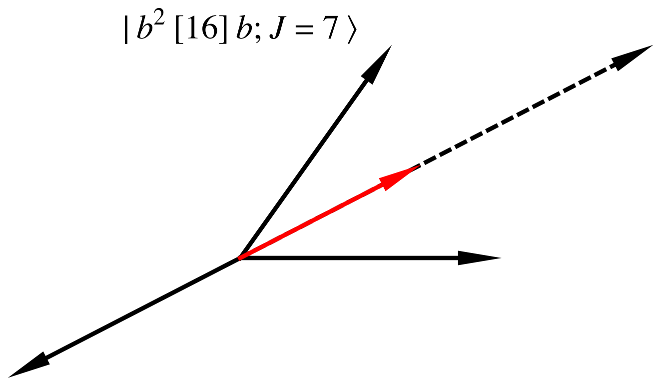

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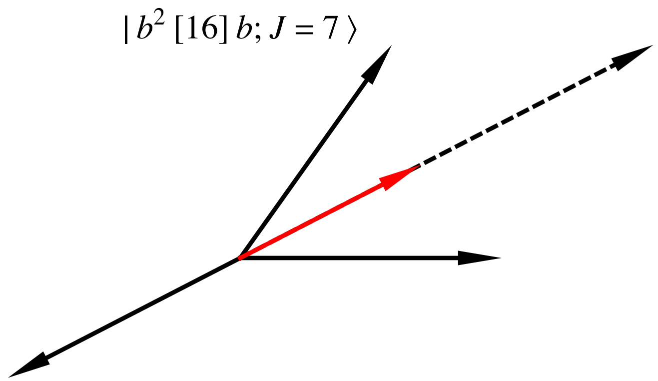

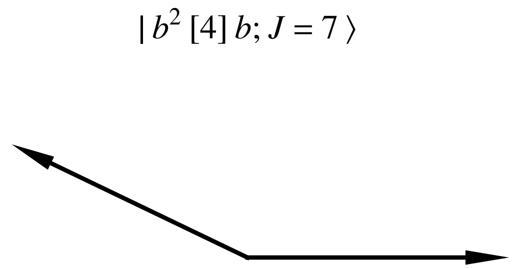

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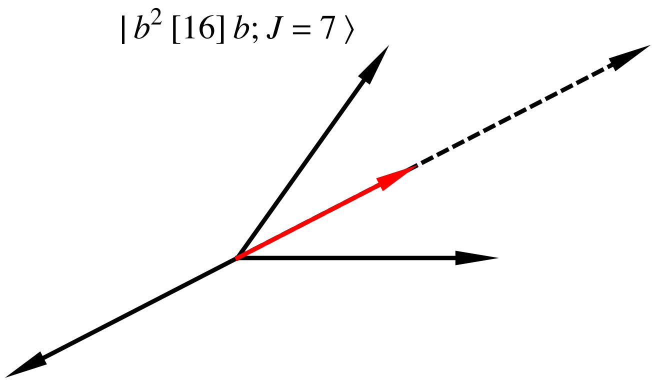
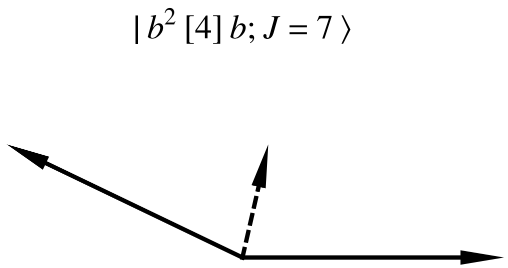
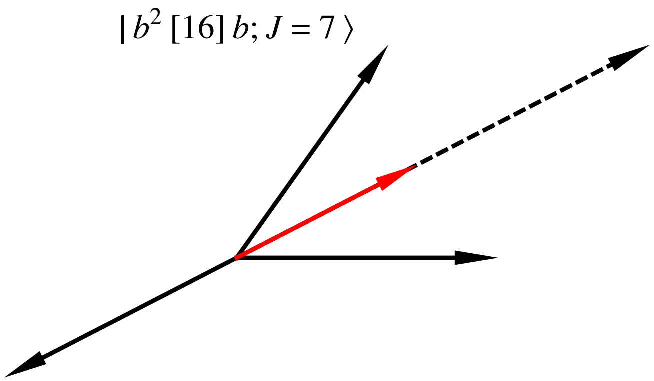
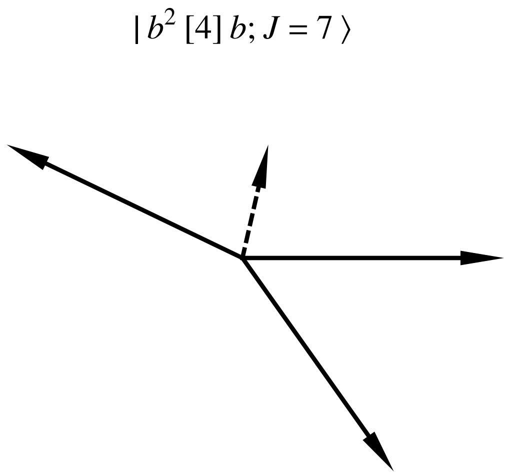




Pitfalls of non-orthogonal bases





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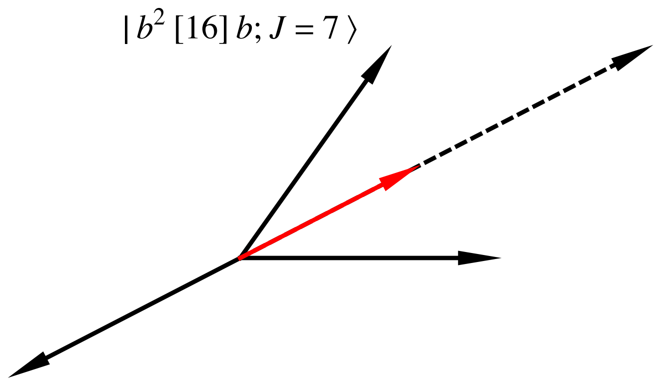

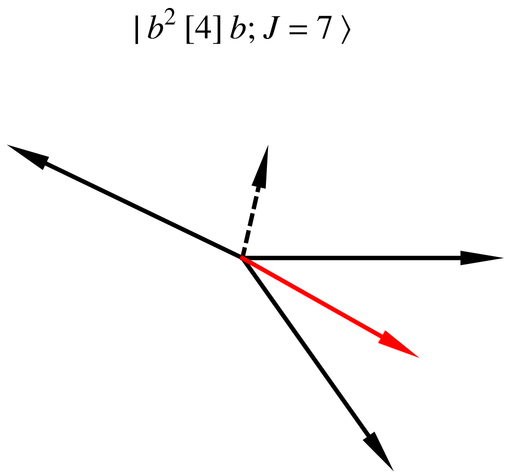

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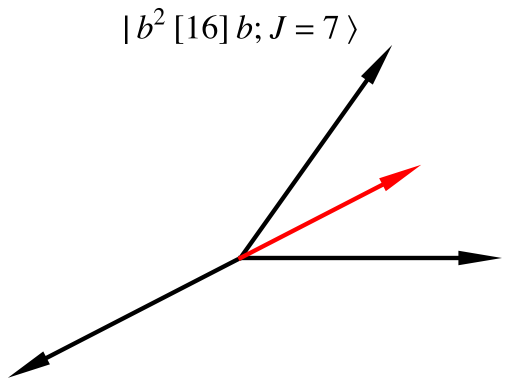

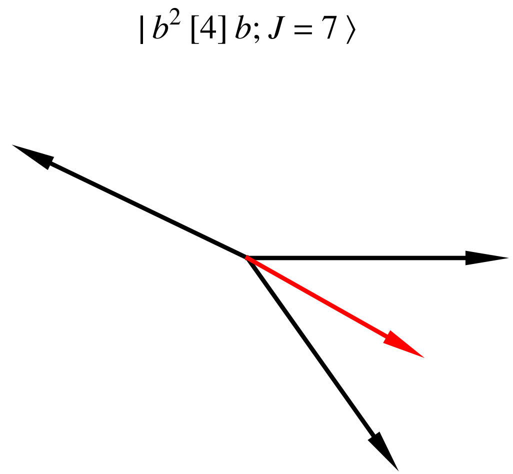

Pitfalls of non-orthogonal bases

In the $1g_{9/2}$ nucleon-pair shell model:

$$\langle B^2[4]\,B;J=7 | B^2[16]\,B;J=7\rangle$$

$$=\sqrt{\frac{112919600563049280}{139849953265085321}}=0.899$$

In the b-IBM:

$$\langle b^2[4]\,b;J=7 | b^2[16]\,b;J=7\rangle=\sqrt{\frac{7012200}{8733503}}=0.896$$

∴ The B pair behaves as a boson.